ICCHP-AAATE 2022 Open Access Compendium "Assistive Technology, Accessibility and (e)Inclusion" Part I

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Linz, 2022

JKU Universitätsbibliothek

Persistent Link: https://doi.org/10.35011/icchp-aaate22-p1-38

urn:nbn:at:at-ubl:3-13118

Universal Design of Inquiry-Based Mathematics Education in Universities

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Abstract. This article context is the teaching and learning of mathematics at university level. The article's author was a member of the team working on the international PLATINUM that aimed to improve learning and teaching of mathematics at universities. The project partners are aware of the fact that the learning of mathematics at universities is very often procedural and students only learn to reproduce mathematical procedures in line with tests and examinations. The project partners aimed to change this trend by designing and implementing inquiry-based activities in order to create the conditions for a deeper conceptual understanding of mathematics for their students so they should have stronger analytical and problem-solving skills. They payed attention to difficulties students with identified needs may encounter in the different phases of an inquiry process. In this article, we offer two examples of teaching units to introduce readers with Universal Design recommendations we designed with respect to students with identified needs so they can actively participate in mathematics courses full of inquiry-based activities.

Keywords: universal design, inquiry-based education, mathematics, university

1 Introduction

The international project PLATINUM[46] was funded by the EU Erasmus+ programme (September 2018 to December 2021). Partners of the project, mathematics lecturers, educators and researchers coming from eight European universities, sought to innovate in their practice as they are aware of the following fact: "Considerable evidence shows that the learning of mathematics widely is highly procedural and not well adapted to using and applying mathematics in science and engineering and the wider world; also, that students learn to reproduce mathematical procedures in line with tests and examinations, rather than developing a relational, applicable, creative view of mathematics that they can use more widely." [1, p. 8] The aim of the project partners was to verify whether an **inquiry-based** approach to teaching and learning will help to change this trend for university students of mathematics.

[46] PLATINUM—Partnership for Learning and Teaching in University Mathematics

The developmental outcomes of the project are presented in the book *Inquiry in University Mathematics Teaching and Learning. The Platinum Project* [1]. As students with identified (or special) needs were one of the three core topics of the project (besides pedagogy and didactics and ICT: new technologies and digital competences), the project partners paid attention to difficulties these students may encounter in the different phases of an inquiry process. “During the first year of the project, they were asked to describe the communication between teachers, students with identified needs and offices for their support in the partners’ universities” [1, p. 50]. They confirmed the information on such students is usually offered at the beginning of the semester, but they develop their courses much earlier. They concluded they should take the needs of their students into account when designing the very processes of learning, assessment and organisation, and not make adjustments on demand with the intention to fit into existing approaches to learning and teaching because the course’s design has already been finalised.

The previous ideas require a tool to design courses well in advance with respect to all the different needs of future students. **Universal Design** is an option suitable for this purpose and the project partners were introduced to this methodological framework. Later on, they tried to develop their courses and inquiry-based teaching units with regard to Universal Design recommendations proposed by the author of this article. This approach is based on the **Social model of disability**, in which school authorities take responsibility and remove barriers to education in order to ensure an inclusive learning environment for as many students as possible.

2 The State of the Art in this Area

Much work has been done recently in the field of inquiry-based mathematics and science education, but mostly at primary and secondary school levels. Let’s name few of European research projects addressing this topic:

- The Fibonacci Project (http://www.fibonacci-project.eu, 2010–2013) contributed to the dissemination of inquiry-based science and mathematics education throughout the European Union. 25 partners from 21 European countries under the supervision of a scientific committee collaborate to transfer inquiry-based methodology to primary and secondary schools in Europe;
- the PRIMAS Project (https://primas-project.eu, 2010–2013) was realised by a consortium of 14 universities from 12 European countries that aimed to promote activities and materials supporting teachers to implement and use inquiry-based learning approaches in their classrooms;
- the PROFILES project (http://www.profiles-project.eu/}, 2010–2014) was focused on inquiry-based approach in science education through raising the self-efficacy of teachers from 20 European countries who worked together in order to implement existing, exemplary context-led teaching materials enhanced by teacher relevant, training and intervention programmes.
- the MaSciL Project (https://mascil-project.ph-freiburg.de, 2012–2016) aimed to make science and mathematics more meaningful to students or primary and

secondary schools. 18 partners from 13 European countries researched how to innovate a teaching culture to include real-life contents from the world of work.

The PLATINUM project is unique as it deals with inquiry-based mathematics education at universities, i.e. in a very different learning environment compared to primary and secondary schools. Lectures with hundreds of students with diverse backgrounds, loaded curricula to be covered in a short period of time, and lack of pedagogical experience—these are the main obstacles for university mathematics teachers to start with inquiry-based approach.

Regarding the universal design of inquiry-oriented mathematics instruction (IBME), we (the project partners) drew on the work of Ronald L. Mace (North Carolina State University). His research group introduced the Universal Design principles in 1985 as a "design of products and environments to be usable by all people, to the greatest extent possible, without the need for adaptation or specialized design" [2]. Later on, **Universal Design for Learning** (UDL) was developed at the Center for Applied Special Technology led by Anne Meyer and David Rose. This framework is closely related to education and aims to improve and optimize teaching and learning for all people.[47] **Accessibility of digital content** is under the care of the World Wide Web Consortium (W3C), international community leading the development of standards on this field. We took all these sources of information into account when designing mathematics courses and its inquiry-based teaching units with respect to students with identified needs.

3 The Methodology Used

When developing Universal Design recommendations for IBME we first needed to understand what pedagogical processes are present during inquiry-based instruction. "Inquiry is about asking questions and seeking answers, recognising problems and seeking solutions, exploring and investigating to find out more about what we do that can help us do it better" [3, p. 396]. During inquiry-based activities, students interact in small groups or together with teachers, examine textbooks and other sources of information to see what is already known, use tools to gather, analyse, and interpret data, make observations, propose explanations and predictions and communicate results of their work [4]. We identified these processes chronologically by breaking down an inquiry activity into some typical sub-activities (see the Figure 1).

Inquiry in university mathematics can be organised at a large hall, a small classroom, a computer or another lab. Students can use computers or manipulate physical instruments. Very often it starts with a *teacher's presentation* of the problem. He/she can also prepare written instructions or *information resources* students are supposed to work with during inquiry. Students usually *collaborate in teams*, they may prepare long-term projects. Very often they are supposed to *use specialized applications* to gather, analyse and interpret data. They *discuss* their findings, observations, they share their thoughts or questions with other students or teachers. Finally they may be asked to *present their results* with others.

[47] https://udlguidelines.cast.org/

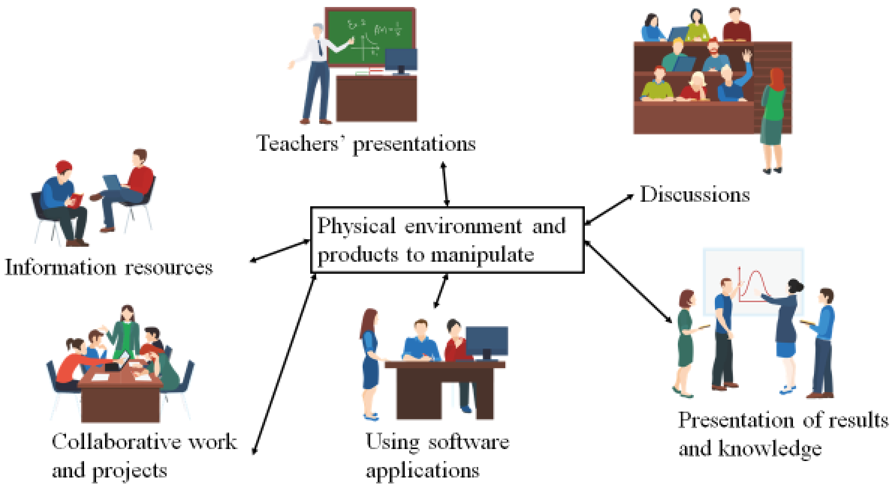

Fig. 1. Pedagogical processes that are present during inquiry-based instruction (Abstract vector created by macrovector_official, http://www.freepik.com)

4 The R&D Work and Results

After dividing IBME into the sub-activities, we

- tried to capture differences of students with identified needs when they undertake any of the previous processes and
- prepared a set of recommendations suitable for teachers who plan to establish an inclusive learning environment during IBME.

We tried to project general principles of Universal Design, Universal Design for Learning and W3C Accessibility Standards[48] onto the inquiry-based education of mathematics at universities. We selected/interpreted the most important ideas of these frameworks and offer them in the form of recommendations relevant to each pedagogical process mentioned above.

In this article, we do not offer a detailed list of the recommendations as it can be found in the book *Inquiry in University Mathematics Teaching and Learning. The Platinum Project* [1] and its Chapter 4. Instead, we demonstrate benefits of the recommendations' implementation on two examples of inquiry-based tasks designed by PLATINUM project partners.

[48] https://www.w3.org/WAI/standards-guidelines/

4.1 Example 1: Universal Design of the task on Complex Numbers

We start with an example given by a colleague from Loughborough University, one of the Platinum project partners. She teaches students of Foundation programme which should prepare them to enter the degree on engineering or science. Many of them are with identified needs. On Figures 2 and 3 there is a task involving inquiry on addition of complex numbers. Students work in pairs in a computer lab with the software called Autograph[49] for helping them to explore complex number arithmetic in a geometric perspective and connect geometric insights with algebraic manipulation. There are two screen shots of the Autograph file (see the Figure 2) and then structured instructions the colleague prepared for her students (see the Figure 3).

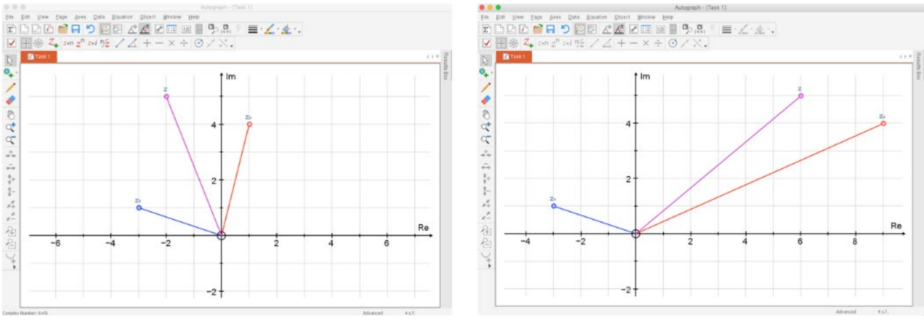

Fig. 2. Two screen shots of the Autograph file with graphical representation of the three complex numbers z_1, z_2 and z given in the Task 1 (the image on the left) and the change of their position if z_2 is moved (the image on the right).

Task 1: There are three complex numbers labelled z_1, z_2 and z; z_1 is to be kept fixed while z_2 and z can be moved. Select z_2 and move it until z reaches the position 6+5j.

1. What complex number is z_2? Right click and "Unhide All" to check your answer. The correct answer appears in green.
2. What is the relationship between z_1, z_2 and z?
3. Now calculate by hand: With z_1=$-3+j$ and z=$6+5j$ find z_2 such that $z_1+z_2=z$.
4. Re-load Task 1. Move z_2 around the screen and notice how z changes as a consequence. What is the geometric connection between z_2, z and the complex number z_1 (which has stayed the same during your movements)?
5. Now you are allowed to move both z_1 and z_2. Move these to different locations but make sure that z still ends up being $6+5j$. Make note of the positions of z_1, z_2. Does your geometric connect from 4. still hold?
6. Repeat another four times so that you have five different pairs of values for z_1 and z_2 with each of them making z to be at $6+5j$. For all of these, what is the relationship between z_1, z_2 and z and does your geometric relationship still hold for each of them?

[49] https://completemaths.com.autograph

Figure 3. Instructions for the original Task 1 in the complex number arithmetic teaching unit, used in the Loughborough Foundation programme [1, p. 110].

The feedback from her students was that the instructions are very detailed and too wordy. Students with dyslexia noticed they lost the track during reading instructions.

1. She reduced the instructions. But still the tasks remain the same, only their presentation is different.
2. She added approximate times for the whole task and for particular subtasks. This was helpful for students with autism spectrum disorder who need to know some boundaries to the time they spend on tasks.
3. She added colors which was useful for students with dyslexia. All the three complex numbers are coloured differently (z_1 in blue, z_2 in red, and z in pink) and this may help to distinguish them more easily (see the Figure 4).

Task 1: (Total time 15–20 mins.)
Open the Autograph file *Task 1*.
There are three complex numbers labelled z_1, z_2 and z; z_1 is to be kept fixed while z_2 and z can be moved. Select z_2 and move it until z reaches the position 6+5*j*.

1. What complex number is z_2?
   Right click and "Unhide All" to check your answer. (2-3 mins.)
2. What is the geometrical relationship between z_1, z_2 and z? (2-3 mins.)
3. Now calculate by hand: With $z_1=-3+j$ and $z=6+5j$ find z_2 such that $z_1+z_2=z$. (2-3 mins.)
4. Re-load Task 1. Move z_2 around the screen and notice how z changes. Describe the position of z in relation to z_1 and z_2. (5 mins.)
5. Explore this relationship. Move z_1 and z_2 to different locations but make sure that z still ends up being $6+5j$. Does what you thought still hold? (5 mins.)

Fig. 4. Modified instructions in the Task 1 about addition of complex numbers, after application of some Universal Design recommendations [1, p. 121].

She helped students to organise their inquiry more easily as she structured the whole task into small subtasks. She enhanced the visual readibility of the instructions (modifications 1, 3), and indicated a time schedule of inquiry (modification 2). These improvements are in line with the Universal design recommendations for Information resources and their preparation [1, p. 63], and useful for all the students not only those who display differences in reading skills, time management, concentration, planning activities, etc.

Using the specialised software Autograph during the inquiry on complex numbers addition is a fundamental part of the students' work. It is therefore helpful to optimise access to this application. We recommend letting the students know in advance about any specialised software they are supposed to work actively during instruction. Sharing suitable support materials to students can save their time during the inquiry as they can familiarise themselves with the interface of the software in advance. If students with visual impairment or physical disabilities are supposed to work actively with such a tool as Autograph, they may need an advice on how to use the software effectively. As some subtasks require students to move graphical objects and change their position in the Autograph workspace, those users who access applications only with keyboard and use key strokes for any mouse action may encounter barriers when trying to use the tool without previous preparation. In such a case a teacher should consult accessibility of applications with experts in assistive technology in order to find the best option for these students [1, p. 64–65].

All these recommendations are based on Universal Design principles but are not sufficient enough for blind students. As the goal of the inquiry is to explore complex number arithmetic in a geometric perspective, students who cannot use their sight are not able to follow visual display of complex numbers in Autograph, manipulate their position and compare arithmetic and graphical representation of these mathematical objects. An individual adjustment is needed in such a case. Teachers should therefore know how to get in touch with institutions responsible for these adjustments and collaborate with experts to deliver them in time and properly [1, p. 67].

4.2 Example 2: Universal Design of the task on Function properties

The second example is related to the course *Mathematical Analysis 1* offered to students at the Faculty of Education, Masaryk University (Brno), who prepare themselves for the career of mathematics teachers in secondary schools. Usually few of them are with identified needs. Students are introduced with the course topics during lectures on a more theoretical level (2 hours per week). Seminars are more practical as their tutors discuss the course key terms with the students and guide them during practical solution of examples (2 hours per week).[50]

The seminar tutors designed a short inquiry-based activity (see below *The task instructions*) intended to encourage the students to

- "recall the concept of limit and continuity of a real function, and
- get back to their own resources and read through the notes they made about the topic during the previous lecture" [1, p. 243].

The task instructions: Make groups of 2–4 people. One specifies limit conditions or requirements on continuity of an unknown function. The others try to find an example of the function which meets the requirements. You can change the roles then.

Examples of requirements:

1. Find the function $f(x)$ such that $\lim_{x\to 3} f(x) = 5$.
2. Find the function $f(x)$ such that $\lim_{x\to 3} f(x) = 5$, but $f(x)$ is not continuous for $x = 3$.

[50] See details in the case study published at [1, p. 242–247].

3. Find the function $f(x)$ such that $\lim_{x\to\infty} f(x) = 0$ (see the graph of the function $f(x) = \arctan x - \frac{\pi}{2}$ corresponding to this condition on the Figure 5).

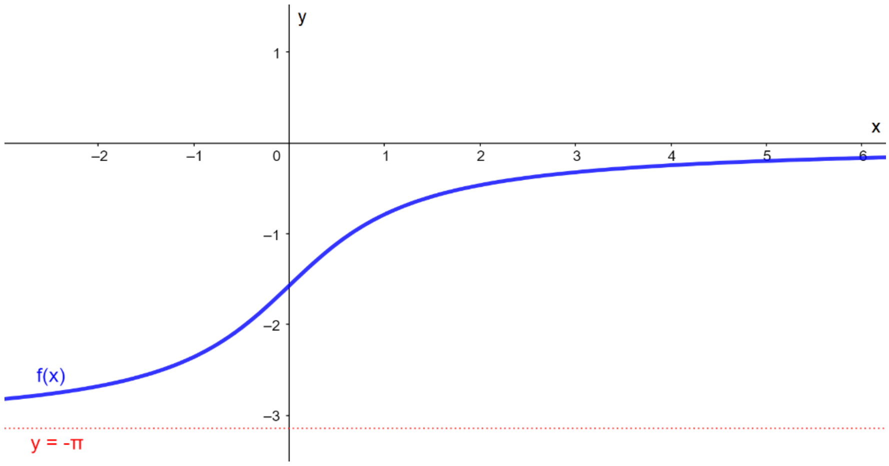

Fig. 5. Graph of the function $f(x) = \arctan x - \frac{\pi}{2}$ such that $\lim_{x\to\infty} f(x) = 0$.

After the first implementation of the activity during the seminar, tutors came with few remarks and proposals of improvement.

- "All students made groups and started actively working on the task" [1, p. 243] as tutors enabled flexible groupings with no fixed requirement on the number of persons per teams and designed the activity as a game in which one "plays" against others trying to come with conditions not easy to satisfy.
- The goal of the activity was not clearly defined. Some students took the examples of requirements listed in the instructions and used them during the activity. That was not the intention of the activity's designers, they gave these examples only for inspiration, not for use.
- The time reserved for team working was not specified, the topic of the following discussion was communicated only verbally and not very precisely, only in a way: "let us know your impression of the activity". As a result, students were not well prepared to contribute to the discussion and came with general comments, in most cases not very valuable.
- Some groups worked slower, therefore not all their members made it to "play" against others. The tutors realised that they should have encouraged their students more strongly to use applications for plotting graphs of functions, which would have saved time in verifying whether their proposal was correct or not.

The designer of the activity reflected on the feedback and re-designed the instructions:

1. He added approximate time for the whole activity.
2. He defined expectations for the group work more clearly. He added the statement asking students to create their own limit or continuity conditions and not use these ones listed in the examples of requirements. He added a note to the instructions in which he recommended to use applications for plotting graphs of functions.
3. In order to prepare the students for the following discussion, he closed the instructions adding one more task: to select, present and comment the most interesting example the group dealt with. It was up to all the teams whether they describe the selected example verbally in front of others or write it on the sheet of paper and share it with the tutor.

All these improvements are based on Universal Design principles. In this case, respecting recommendations for collaborative work in teams (see [1, p. 63–64]) and discussions (see [1, p. 66]) helps the students, but also the tutors who can better realise their ideas about the students' work corresponding to the objectives with which they designed learning activities.

The seminar is held in a classroom not equipped with computers. As some planned learning activities require use of computer-based devices, tutors ask students to bring their laptops, tablets or at least mobile phones to seminar sessions on a regular basis. As for the previous Example 1 on visual manipulation with complex numbers in Autograph, the students should familiarise themselves in advance with the interface of the software for plotting graphs of functions (Wolfram Alpha, Geogebra, etc.). Again, according to Universal design principles, tutors should let the students know about suitable support materials and consult with experts in assistive technology to determine which of these tools is appropriate for users with visual impairment or physical disability [1, p. 64–65].

5 The Scientific and Practical Impact or Contributions to the Field

The main objective of the PLATINUM project is to innovate university mathematics education. We established an international group and local communities of mathematics teachers, educators, and researchers who use an inquiry-based approach to create the conditions for a deeper conceptual understanding of mathematics for their students so they should have stronger analytical and problem-solving skills. Designing inquiry-based teaching units, implementing them during instruction, and evaluating their effectivity needs a closer collaboration of all the members of such communities. It is not only the students who do inquiry, but also their teachers and educators who analyse the IBME data, make observations and reflections and help improve the design of inquiry-based activities. This developmental research approach and three-layer model (students, teachers, educators/researchers) was adopted by the PLATINUM partners within their local communities and helps to facilitate the design and implementation of IBME.

By applying Universal Design principles we hope for creating an inclusive inquiry-based learning environment that reaches the needs of as many students as possible.

6 Conclusion

In this article, we try to describe results of the European project called PLATINUM. We introduce readers with Universal Design recommendations we designed with respect to students with identified needs so they can actively participate in mathematics courses full of inquiry-based activities. Collaboration on the design and implementation of these recommendations can be based on principles of developmental research and three-layer model described in [1], [3] or [4]. “While students undertake inquiry-based instruction, teachers inquire how to implement some of the Universal Design ideas into their lectures and seminars. Such development is continuous and clearly needs the feedback not only from students but also from experts on inclusive education in order to evaluate the effectiveness of implemented recommendations and plan other modifications of the course” [1].

As the project is unique and no one before us dealt with inquiry-based mathematics education at university level and within such a broad international group, a change in improving teaching mathematics in tertiary level can be a long and slow ‘journey’. We hope that the PLATINUM project and its results will contribute to the above mentioned goals.

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